Collective properties of magnetobiexcitons in quantum wells and graphene superlattices

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The Bose-Einstein condensation and superfluidity of quasi-two-dimensional spatially indirect magnetobiexcitons in a slab of superlattice with alternating electron and hole layers consisting from the semiconducting quantum wells (QWs) and graphene superlattice in high magnetic field are considered. The two different Hamiltonians of a dilute gas of magnetoexcitons with a dipole-dipole repulsion in superlattices, consisting of both QWs and graphene layers (GLs) in the limit of high magnetic field, have been reduced to one effective Hamiltonian—a dilute gas of two-dimensional excitons with the renormalized effective mass of the magnetoexciton, which depends on the magnetic field. This Hamiltonian does not include the vector potential. Moreover, for N excitons we have reduced the problem of $2N\times 2$-dimensional space onto the problem of $N\times 2$-dimensional space by integrating over the coordinates of the relative motion of an electron and a hole. The instability of the ground state of the system of interacting two-dimensional indirect magnetoexcitons in a slab of superlattice with alternating electron and hole layers in high magnetic field is found. The stable system of indirect quasi-two-dimensional magnetobiexcitons, consisting of a pair of indirect excitons with opposite dipole moments, is considered. The density of the superfluid component $n_s(T)$ and the temperature of the Kosterlitz-Thouless phase transition to the superfluid state in the system of two-dimensional indirect magnetobiexcitons, interacting as electrical quadrupoles, are obtained for both the QW and graphene realizations.

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I. INTRODUCTION

The many-particle systems of the spatially indirect excitons in coupled quantum wells (CQWs) in the magnetic field B, as well as without magnetic field, have been the subject of recent experimental investigations.[1–4] These systems are of interest, in particular, in connection with the possibility of the Bose-Einstein condensation (BEC) and superfluidity of indirect excitons or electron-hole pairs, which would manifest itself in the CQW as persistent electrical currents in each well, and also through coherent optical properties and Josephson phenomena.[5–9] In strong magnetic fields ($B>7$ T) two-dimensional (2D) excitons survive in a substantially wider temperature region, as the exciton binding energies increase with magnetic field.[10–16] The problem of the essential interest is also collective properties of magnetoexcitons in high magnetic fields in superlattices and layered system.[17]

In this paper we propose a new physical realization of the magnetoexcitonic BEC and superfluidity in superlattices with alternating electronic and hole layers, that is in a sense representing array of the CQWs or GLs with spatially separated electrons (e) and holes (h) in high magnetic field. Recent technological advances have allowed the production of graphene, which is a 2D honeycomb lattice of carbon atoms that form the basic planar structure in graphite.[18,19] Graphene has been attracting a great deal of experimental and theoretical attention because of unusual properties in its bandstructure.[20–23] It is a gapless semiconductor with massless electrons and holes, which have been described as Dirac-fermions.[24] Since there is no gap between the conduction and valence bands in graphene without magnetic field, the screening effects result in the absence of excitons in graphene without magnetic field. A strong magnetic field produces a gap since the energy spectrum formed by Landau levels becomes discrete. The gap reduces screening and leads to the formation of the magnetoexcitons. We also consider magnetoexcitons in the superlattices with alternating electronic and hole GLs. We suppose that recombination times may be much greater than relaxation times τ_r due to small overlapping of the spatially separation of e- and h-wave functions in the CQW or GLs. In this case electrons and holes are characterized by different quasiequilibrium chemical potentials. Therefore, in the system of indirect excitons in superlattices, as well as in CQW,[5,9] the quasiequilibrium phases appear.[25] While coupled-well structures with spatially separated electrons and holes are typically considered to be under applied electric field, which separates electrons and holes in different quantum wells,[2,3] we assume that there are no external electric fields applied to a slab of superlattice. If "electron" and "hole" quantum wells alternate, there are excitons with parallel dipole moments in one pair of wells, but dipole moments of excitons in another neighboring pairs of neighboring wells have an opposite direction, which causes the attraction between the excitons with opposite dipole moments. This fact leads to the essential distinction of properties of $e-h$ system in superlattices and coupled quantum wells with spatially separated electrons and holes, where indirect exciton system is stable due to the dipole-dipole repelling of all excitons. This difference manifests itself already beginning from three-layer $e-h-e$ or $h-e-h$ system.

In this paper we reduce the problem of magnetoexcitons in the QWs and GLs superlattices in the strong magnetic field to the problem of excitons by deriving an effective Hamiltonian with renormalized mass. It is shown that the instability of the ground state of the system of interacting indirect excitons in the slab of superlattice with alternating e and h layers is established in the strong magnetic field. Two-dimensional indirect magnetobiexcitons, consisting of the indirect magnetoexcitons with opposite dipole moments, are considered in the strong magnetic field. The radius and the binding energy of the indirect magnetobiexciton are calculated. These magnetobiexcitons repel as electrical quadrupoles at long distances. As a result, the system of the indirect magnetobiexcitons is stable. The collective spectrum of the weakly interacting by the quadrupole law two-dimensional indirect magnetobiexcitons is considered in the ladder approximation. The superfluid density $n_s(T)$ of interacting two-dimensional indirect magnetobiexcitons in superlattices is calculated at low temperatures T. We analyze the dependence of the Kosterlitz-Thouless transition temperature,[26] as well as superfluid density on magnetic field.

This paper is organized as follows. In Sec. II, we derive an effective Hamiltonian for magnetoexcitons in both CQWs and two graphene layers in strong magnetic field. In Sec. III, we prove the instability of dipole magnetoexcitons in the QWs and graphene superlattices due to the attraction of oppositely directed dipoles. The superfluidity of quadrupole magnetobiexcitons in the QWs and graphene superlattices is analyzed and discussed in Sec. IV. The results of calculations and discussion are presented in Sec. V. Conclusions follow in Sec. VI.

II. EFFECTIVE HAMILTONIAN FOR MAGNETOEXCITONS IN STRONG MAGNETIC FIELD

A. Hamiltonian for magnetoexcitons in the CQWs

Let's start with two interacting excitons in the CQW in the presence of the external magnetic field $\mathbf{B}$, and for the description of the motion of the exciton center of mass and relative motion of the electron and hole with the masses m_e and m_h and coordinates $\mathbf{r}_e$ and $\mathbf{r}_h$ along the QW, introduce the standard set of Jacobi coordinates

$$\mathbf{R} = \frac{m_e\mathbf{r}_e + m_h\mathbf{r}_h}{m_e + m_h},$$

$$\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h. \tag{1}$$

Without loss of a generality we can take the magnetic field to be in the z direction and it is convenient to work with the symmetric gauge for electrons and holes vector potential $\mathbf{A}_{e(h)}=1/2[\mathbf{B}\times\mathbf{r}_{e(h)}]$. Using these notations and coordinates [Eq. (1)] the Hamiltonian H for 2D spatially separated excitons in magnetic field can be written as

$$\hat{H} = \int d\mathbf{R}\int d\mathbf{r}[\hat{\psi}^{\dagger}(\mathbf{R},\mathbf{r})\hat{\mathcal{H}}\hat{\psi}(\mathbf{R},\mathbf{r})] + \frac{1}{2}\int d\mathbf{R}_1\int d\mathbf{r}_1\int d\mathbf{R}_2\int d\mathbf{r}_2\hat{\psi}^{\dagger}(\mathbf{R}_1,\mathbf{r}_1)\hat{\psi}^{\dagger}(\mathbf{R}_2,\mathbf{r}_2) \times \sum_{i,j=e,h} U^{ij}(\mathbf{r}_{i1}-\mathbf{r}_{j2})\hat{\psi}(\mathbf{R}_2,\mathbf{r}_2)\hat{\psi}(\mathbf{R}_1,\mathbf{r}_1), \tag{2}$$

where $\hat{\mathcal{H}}$ is the operator of an isolated electron-hole pair given by

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 - \frac{e^2}{\epsilon\sqrt{(\mathbf{r}_e-\mathbf{r}_h)^2+D^2}}, \tag{3}$$

with the operator $\hat{\mathcal{H}}_0$ for the noninteracting electron-hole pair in magnetic field presented by

$$\hat{\mathcal{H}}_0 = \sum_{i=e,h}\left[\frac{1}{2m_i}\left(\mathbf{p}_i + \frac{e}{c}\mathbf{A}_i\right)^2\right], \tag{4}$$

In Eqs. (2)–(4) $\hat{\psi}^{\dagger}(\mathbf{R},\mathbf{r})$ and $\hat{\psi}(\mathbf{R},\mathbf{r})$ are the creation and annihilation operators for magnetoexcitons, $\mathbf{r}_e$ and $\mathbf{r}_h$ are two-dimensional vectors of coordinates of an electron and hole, respectively, D is the distance between electron and hole quantum wells, e is the charge of an electron, c is the speed of light, and ϵ is a dielectric constant. In Eq. (2) for electrons and holes from two different excitons, we use the two-particle potentials U^{ij} for the electron-electron, hole-hole, electron-hole, and hole-electron interaction: $U^{ee}(\mathbf{r}_{e1}-\mathbf{r}_{e2})=e^2/(\epsilon|\mathbf{r}_{e1}-\mathbf{r}_{e2}|)$, $U^{hh}(\mathbf{r}_{h1}-\mathbf{r}_{h2})=e^2/(\epsilon|\mathbf{r}_{h1}-\mathbf{r}_{h2}|)$, $U^{eh}(\mathbf{r}_{e1}-\mathbf{r}_{h2})=-e^2/(\epsilon\sqrt{|\mathbf{r}_{e1}-\mathbf{r}_{h2}|^2+D^2})$, $U^{he}(\mathbf{r}_{h1}-\mathbf{r}_{e2})=-e^2/(\epsilon\sqrt{|\mathbf{r}_{h1}-\mathbf{r}_{e2}|^2+D^2})$.

A conserved quantity for an isolated electron-hole pair in magnetic field B is the exciton generalized momentum $\hat{\mathbf{P}}$ defined as

$$\hat{\mathbf{P}} = -i\hbar\nabla_e - i\hbar\nabla_h + \frac{e}{c}(\mathbf{A}_e - \mathbf{A}_h) - \frac{e}{c}[\mathbf{B}\times(\mathbf{r}_e-\mathbf{r}_h)] \tag{5}$$

for the Dirac equation in the GLs,[27] as well as for the Schrödinger equation in the CQWs.[10,12,28]

The Hamiltonian [Eq. (4)] of a single isolated magnetoexciton is commutated with $\hat{\mathbf{P}}$, and hence, they have the same eigenfunctions, which have the following form (see Refs. 10 and 28):

$$\Psi_{k\mathbf{P}}(\mathbf{R},\mathbf{r}) = \exp\left\{i\frac{\mathbf{R}}{\hbar}\left(\mathbf{P} + \frac{e}{c}\mathbf{B}\times\mathbf{r}\right) + i\gamma\frac{\mathbf{P}\mathbf{r}}{2\hbar}\right\}\tilde{\Phi}_k(\mathbf{P},\mathbf{r}), \tag{6}$$

where $\tilde{\Phi}_k(\mathbf{P},\mathbf{r})$ is a function of the internal coordinates $\mathbf{r}$ and the eigenvalue $\mathbf{P}$ of the generalized momentum, and k represents the quantum numbers of the exciton internal motion. The wave function $\tilde{\Phi}(\mathbf{P},\mathbf{r})$ is provided in Refs. 10, 14, and 28:

$$\tilde{\Phi}_k(\mathbf{P},\mathbf{r}) = \Phi_{n_1,n_2}(\mathbf{P},\mathbf{r}) = (2\pi)^{-1/2}2^{-|m|/2}\frac{\tilde{n}!}{\sqrt{n_1!\,n_2!}}\frac{1}{r_B}\,\mathrm{sgn}(m)^m\frac{r^{|m|}}{r_B^{|m|}} \times\exp\left[-im\phi - \frac{r^2}{4r_B^2}\right]L_{\tilde{n}}^{|m|}\left(\frac{r^2}{2r_B^2}\right), \tag{7}$$

where $L_{\tilde{n}}^{|m|}$ denotes Laguerre polynomials, $m=n_1-n_2$, $\tilde{n}=\min(n_1,n_2)$, and $\mathrm{sgn}(m)^m\to 1$ for $m=0$. In high magnetic fields the magnetoexcitonic quantum numbers $k=\{n_+,n_-\}$ for an electron in the Landau level n_+ and a hole in level n_- and $\gamma=(m_h-m_e)/(m_h+m_e)$.

B. Hamiltonian for magnetoexcitons in the bilayer graphene

If the Coulomb interaction between the electron and hole is neglected, the four-component Hamiltonian of electron-hole pairs in the bilayer graphene with spatially separated electrons and holes in one valley in magnetic field B can be obtained by substituting in Eq. (2) the following four-component Hamiltonian $\hat{\mathcal{H}}_0$ for an isolated electron-hole pair:[27]

$$\hat{\mathcal{H}}_0 = v_F \begin{pmatrix} 0 & p_{ex}+ip_{ey} & 0 & 0 \\ p_{ex}-ip_{ey} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{hx}-ip_{hy} \\ 0 & 0 & p_{hx}+ip_{hy} & 0 \end{pmatrix}, \tag{8}$$

where

$$\mathbf{p}_e = -i\hbar\nabla_e + \frac{e}{c}\mathbf{A}_e, \quad \mathbf{p}_h = -i\hbar\nabla_h - \frac{e}{c}\mathbf{A}_h. \tag{9}$$

In Eq. (8) $v_F=\sqrt{3}at/(2\hbar)$ is the Fermi velocity of electrons in graphene, where $a=2.566$ Å is a lattice constant, and $t\approx 2.71$ eV is the overlap integral between the nearest carbon atoms.[29] It should be mentioned that for the bilayer graphene the terms corresponding to the Coulomb attraction between electron and hole in an isolated pair and dipole-dipole interaction between two different electron-hole pairs are the same as for the CQWs and are given by Eqs. (3) and (2), respectively.

The wave function $\tilde{\Phi}(\mathbf{P},\mathbf{r})$ of the relative coordinate for e and h spatially separated in the different GLs can be expressed in terms of the two-dimensional harmonic oscillator eigenfunctions $\Phi_{n_1,n_2}(\mathbf{r})$ given by Eq. (7) ($\gamma=0$). For an electron in Landau level n_+ and a hole in level n_-, the four-component spinor wave function of the relative coordinate is[27]

$$\tilde{\Phi}_{n_+,n_-}(\mathbf{P},\mathbf{r}) = (\sqrt{2})^{\delta_{n_+,0}+\delta_{n_-,0}-2} \begin{pmatrix} s_+s_-\Phi_{|n_+|-1,|n_-|-1}(\mathbf{P},\mathbf{r}) \\ s_+\Phi_{|n_+|-1,|n_-|}(\mathbf{P},\mathbf{r}) \\ s_-\Phi_{|n_+|,|n_-|-1}(\mathbf{P},\mathbf{r}) \\ \Phi_{|n_+|,|n_-|}(\mathbf{P},\mathbf{r}) \end{pmatrix}. \tag{10}$$

The corresponding energy of the electron-hole pair $E^{(0)}_{n_+,n_-}$, which is the eigenvalue of the Hamiltonian (8) is given by[27]

$$E^{(0)}_{n_+,n_-} = \frac{\hbar v_F}{r_B}\sqrt{2}[\mathrm{sgn}(n_+)\sqrt{|n_+|} - \mathrm{sgn}(n_-)\sqrt{|n_-|}], \tag{11}$$

where $s_\pm=\mathrm{sgn}(n_\pm)$ and $r_B=\sqrt{c\hbar/(eB)}$ is a magnetic length.

C. Effective Hamiltonian for magnetoexcitons

In a strong magnetic field at low densities, $n\ll r_B^{-2}$, indirect magnetoexcitons repel as parallel dipoles, and we have for the pair interaction potential

$$U(|\mathbf{R}_1-\mathbf{R}_2|) \equiv U^{ee}+U^{hh}+U^{eh}+U^{he} \simeq \frac{e^2D^2}{\epsilon|\mathbf{R}_1-\mathbf{R}_2|^3}. \tag{12}$$

Let's expand the magnetoexciton field operators in terms of a single magnetoexciton basis set $\Psi_{k\mathbf{P}}(\mathbf{R},\mathbf{r})$:

$$\hat{\psi}^\dagger(\mathbf{R},\mathbf{r}) = \sum_{k\mathbf{P}} \Psi^*_{k\mathbf{P}}(\mathbf{R},\mathbf{r})\hat{a}^\dagger_{k\mathbf{P}},$$

$$\hat{\psi}(\mathbf{R},\mathbf{r}) = \sum_{k\mathbf{P}} \Psi_{k\mathbf{P}}(\mathbf{R},\mathbf{r})\hat{a}_{k\mathbf{P}}, \tag{13}$$

where $\hat{a}^\dagger_{k\mathbf{P}}$ and $\hat{a}_{k\mathbf{P}}$ are the corresponding creation and annihilation operators of a magnetoexciton in $(k,\mathbf{P})$ space and substitute the expansions for the field creation and annihilation operators into Eq. (2). Due to the orthonormality of the wave functions $\Phi_{n_+,n_-}(\mathbf{0},\mathbf{r})$ the projection of the Hamiltonian [Eq. (2)] onto the lowest Landau level results in the effective Hamiltonian, which does not reflect the spinor nature of the four-component magnetoexcitonic wave functions in graphene. Since typically, the value of r is r_B, and $P\ll\hbar/r_B$ in this approximation, the effective Hamiltonian $\hat{H}_{\mathrm{eff}}$ in the magnetic momentum representation P in the subspace the lowest Landau level, which are $n_+=n_-=0$ for QWs, and $n_+=n_-=1$ for GLs, has the same form (compare with Ref. 9) as for the two-dimensional boson system without a magnetic field, but with the magnetoexciton magnetic mass m_B (which depends on B and D; see below) instead of the exciton mass $(M=m_e+m_h)$, magnetic momenta instead of ordinary momenta:

$$\hat{H}_{\mathrm{eff}} = \sum_{\mathbf{P}} \varepsilon_0(P)\hat{a}^\dagger_{\mathbf{P}}\hat{a}_{\mathbf{P}} + \frac{1}{2}\sum_{\mathbf{P}_1,\mathbf{P}_2,\mathbf{P}_3,\mathbf{P}_4} \langle \mathbf{P}_1,\mathbf{P}_2|\hat{U}|\mathbf{P}_3,\mathbf{P}_4\rangle \hat{a}^\dagger_{\mathbf{P}_1}\hat{a}^\dagger_{\mathbf{P}_2}\hat{a}_{\mathbf{P}_3}\hat{a}_{\mathbf{P}_4}, \tag{14}$$

where the matrix element $\langle \mathbf{P}_1,\mathbf{P}_2|\hat{U}|\mathbf{P}_3,\mathbf{P}_4\rangle$ is the Fourier transform of the pair interaction potential $U(R)=e^2D^2/\epsilon R^3$, and for the lowest Landau level we denote the spectrum of the single exciton $\varepsilon_0(P)\equiv\varepsilon_{00}(\mathbf{P})$. For an isolated magnetoexciton on the lowest Landau level at the small magnetic momenta under consideration, $\varepsilon_0(\mathbf{P})\approx P^2/(2m_B)$, where m_B is the effective *magnetic* mass of a magnetoexciton in the lowest Landau level and is a function of the distance D between e and h layers and magnetic field B (see Ref. 14). In strong magnetic fields at $D\gg r_B$ the exciton magnetic mass is $m_B(D)=\epsilon D^3/(e^2r_B^4)$ for the QWs (Ref. 14) and $m_B(D)=\epsilon D^3/(4e^2r_B^4)$ for the graphene layers.[30] The projection of the electron-hole Hamiltonian in magnetic field for the CQWs [Eq. (2)] and GLs [Eq. (8)] onto the lowest Landau level results in the effective Hamiltonian [Eq. (14)] with renormalized mass and where term related to the vector potential is missing. The magnetic field in the effective Hamiltonian [Eq. (14)] is present in the renormalized mass of the magnetoexciton m_B. Therefore, Hamiltonian for the spatially separated electrons and holes in two-layer system [Eq. (2)] with the operator of the kinetic energy for an isolated electron-hole pair $\hat{\mathcal{H}}_0$ defined by Eq. (4) for the CQWs and Eq. (8) for the bilayer graphene can be reduced in high magnetic field to the effective Hamiltonian [Eq. (14)]. Magnetic field B is reflected by the effective Hamiltonian [Eq. (14)] only through the effective *magnetic* mass of a magnetoexciton m_B in the expression for $\varepsilon_0(P)$ in the first term of $\hat{H}_{\text{eff}}$. The only difference in the effective Hamiltonian [Eq. (14)] for the CQWs and bilayer graphene realizations of two-layer systems is that m_B for the bilayer graphene is four times less than for the CQWs due to the four-component spinor structure of the wave function of the relative motion for the isolated noninteracting electron-hole pair in magnetic field [Eq. (10)].

Transitions between Landau levels due to the Coulomb electron-hole attraction for the large electron-hole separation $D \gg r_B$ can be neglected if the following condition is valid: $E_b = e^2/(\epsilon_b D) \ll \hbar\omega_c = \hbar eB(m_e+m_h)/(2m_e m_h c)$ for the QWs and $E_b = 4e^2/(\epsilon D) \ll \hbar v_F/r_B$ for the GLs, where E_b and ω_c are the magnetoexcitonic binding energy and the cyclotron frequency, respectively. This corresponds to the high magnetic field B, the large interlayer separation D, and large dielectric constant of the insulator layer between the graphene layers. As it was defined previously, here $v_F = \sqrt{3}at/(2\hbar)$ is the Fermi velocity of electrons.

It was shown in Refs. 31 and 32 that taking into account the spin degree of freedom can qualitatively modify the results for exciton-polariton condensation at magnetic fields lower than critical magnetic field. We assume that magnetic field B considered in this paper is above the critical one, and, therefore, Zeeman splitting does not effect on the spectrum of collective excitations according to Fig. 1 in Ref. 31.

III. INSTABILITY OF DIPOLE MAGNETOEXCITONS IN QW AND GRAPHENE SUPERLATTICES

Let us show that the low-density system of weakly interacting two-dimensional indirect magnetoexcitons in superlattices is instable, contrary to the two-layer system in the CQW. At the small densities of $nr_B^2 \ll 1$ the system of indirect excitons at low temperatures is the two-dimensional weakly nonideal Bose gas with normal to wells dipole moments $\mathbf{d}$ in the ground state ($d = eD$, D is the interwell separation), increasing with the distance D between wells. In contrast to ordinary excitons, for the low-density spatially indirect magnetoexciton system the main contribution to the energy is originated from the dipole-dipole interactions U_- and U_+ of magnetoexcitons with opposite (see Fig. 1) and parallel dipoles, respectively. The potential energy of interaction between two indirect magnetoexcitons with parallel $U_+(R)$ and opposite $U_-(R)$ dipoles is a function of the distance R between indirect magnetoexcitons along the quantum wells or graphene layers:

$$U_+(R) = \frac{2e^2}{\epsilon R} - \frac{2e^2}{\epsilon\sqrt{R^2+D^2}},$$

$$U_-(R) = \frac{e^2}{\epsilon R} - \frac{2e^2}{\epsilon\sqrt{R^2+D^2}} + \frac{e^2}{\epsilon\sqrt{R^2+4D^2}}. \qquad (15)$$

The behavior of the potential energies $U_+(R)$ and $U_-(R)$ as the functions of the distance between two excitons R is shown in Fig. 2. We suppose that $D/R \ll 1$ and $L/R \ll 1$, where L is the mean distance between dipoles normal to the wells. We consider the case, when the number of quantum wells k in superlattice is restricted $k \ll 1/(D\sqrt{\pi n})$. This is valid for small k or for sufficiently low exciton density.

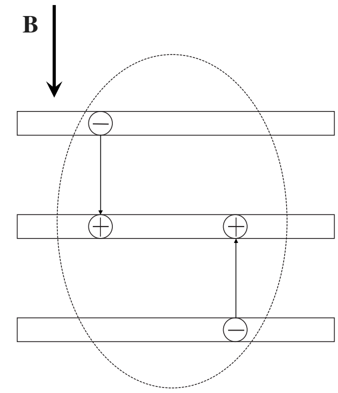

FIG. 1. Two-dimensional indirect magnetobiexcitons consisting of indirect magnetoexcitons with opposite dipole moments, located in neighboring pairs of QWs or graphene layers.

The distinction between magnetoexcitons and bosons manifests itself in exchange effects.[9,33] The exchange interaction in the spatially separated system is suppressed in contrast to the $e-h$ system in one well due to smallness of the tunnel exponent T connected with the penetration through barrier of the dipole-dipole interaction. Hence, at $D \gg r_B$ exchange phenomena, connected with the distinction between excitons and bosons, can be neglected for both the QWs and graphene layers.[30] Two indirect exciton in a dilute system interact according to Eq. (15), where R is the distance between exciton dipoles along the QWs and GLs. Small tunneling parameter connected with this barrier is[30]

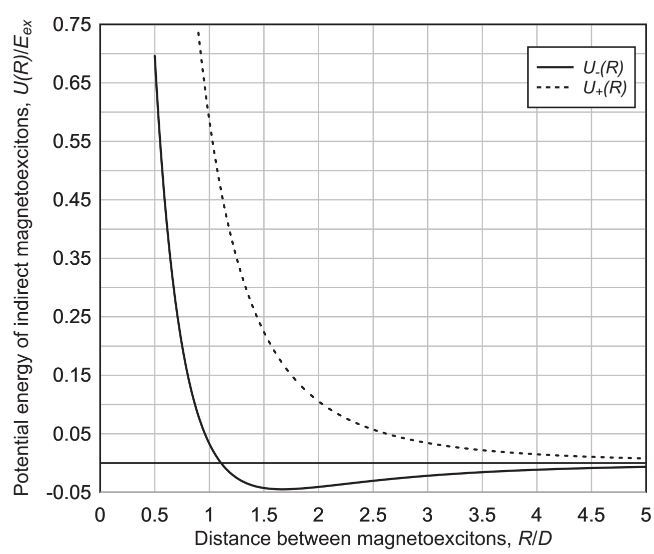

FIG. 2. The potential energy of the interaction of indirect magnetoexcitons with parallel $U_+(R)$ and opposite $U_-(R)$ dipoles, located in neighboring pairs of QWs or GLs (in units of the binding energy of the indirect magnetoexciton $E_{ex} = e^2/\epsilon D$), as a function of the distances R between magnetoexcitons along the QWs or GLs (in units of D).

$$\exp\left\{-\frac{1}{\hbar}\int_{r_0}^{R_0}\sqrt{2m_B\left[U_{+(-)}-\frac{\kappa^2}{2m_B}\right]}dR\right\},$$

where the characteristic momentum of the system is given by $\kappa=\sqrt{2m_B\mu_d}$ [μ_d is the chemical potential of the very dilute system of magnetoexcitons, which is in a dilute 2D weakly nonideal Bose gas very small and proportional to $n/\log n$ (Ref. 34)], r_0 is the 2D radius of magnetoexciton, R_0 is the classical turning point for the dipole-dipole interaction determined from the condition $U_{+(-)}(R_0)=\mu_d$. Hence, for the parallel dipoles R_0 is given by $R_0=(2m_Be^2D^2/\kappa^2)^{1/3}$. Since μ_d is very small, for the opposite dipole the classical turning point can be approximated by the condition $U_-(R_0)=0$ and $R_0\approx D$ (see Fig. 2). In high magnetic fields the small parameter mentioned above has the form $\exp[-2\hbar^{-1}(m_B)^{1/2}eDr_0^{-1/2}]$. So at zero temperature $T=0$ the dilute gas of magnetoexcitons, which is a boson system, form the Bose-Einstein condensate.[35,37] Therefore, the system of indirect magnetoexcitons can be treated by the formalism applicable for a boson system.

For the analysis of a stability of the ground state of the weakly nonideal Bose gas of indirect excitons in superlattices, let us apply the Bogoliubov approximation. The total Hamiltonian $\hat{H}_{\rm tot}$ of the low-density system of the indirect magnetoexcitons in the superlattice, which is the generalization of the effective Hamiltonian of magnetoexcitons in a two-layer system [Eq. (14)], is given by

$$\hat{H}_{\rm tot}=\hat{H}_0+\hat{H}_{\rm int}. \tag{16}$$

Here $\hat{H}_0$ is the effective Hamiltonian of the system of noninteracting magnetoexcitons:

$$\hat{H}_0=\sum_{\mathbf{P}}\varepsilon_0(p)(a^+_{\mathbf{P}}a_{\mathbf{P}}+b^+_{\mathbf{P}}b_{\mathbf{P}}+a^+_{-\mathbf{P}}a_{-\mathbf{P}}+b^+_{-\mathbf{P}}b_{-\mathbf{P}}). \tag{17}$$

In Eq. (17) $\varepsilon_0(p)=p^2/(2m_B)$ is the spectrum of isolated two-dimensional indirect magnetoexciton, $\mathbf{P}$ represents the excitonic magnetic momentum, $a^+_{\mathbf{P}}$, $a_{\mathbf{P}}$, $b^+_{\mathbf{P}}$, $b_{\mathbf{P}}$ are creation and annihilation operators of the magnetoexcitons with up and down dipoles. The Hamiltonian which describes the interaction between magnetoexcitons is

$$\hat{H}_{\rm int}=(2S)^{-1}\sum\nolimits_{\mathbf{P}_1+\mathbf{P}_2=\mathbf{P}_3+\mathbf{P}_4}[U_+(a^+_{\mathbf{P}_4}a^+_{\mathbf{P}_3}a_{\mathbf{P}_2}a_{\mathbf{P}_1}+b^+_{\mathbf{P}_4}b^+_{\mathbf{P}_3}b_{\mathbf{P}_2}b_{\mathbf{P}_1}) - U_-a^+_{\mathbf{P}_4}b^+_{\mathbf{P}_3}a_{\mathbf{P}_2}b_{\mathbf{P}_1}], \tag{18}$$

where U_+ and U_- are the 2D Fourier images of $U_+(R)$ and $U_-(R)$ at $P=0$, respectively, and S is the surface of the system. Let us mention that the appropriate cutoff parameter for this Fourier transform is the classical turning point of the dipole-dipole interaction. Note that the cutoff parameter R_0 for the potential $U_+(R)$ is much greater than for $U_-(R)$ [the cutoff parameters R_0 for the both potentials can be represented in Fig. 2 by the points where the curves corresponding to $U_+(R)$ and $U_-(R)$ are crossed by the chemical potential μ_d represented by the horizontal straight line placed right above but close to $U_{+(-)}(R)=0$]. Therefore, we claim that $U_+>0$, $U_-<0$, and $|U_-|>|U_+|$.

The physical meaning of the terms of the type $a^+_{\mathbf{P}_4}b^+_{\mathbf{P}_3}a_{\mathbf{P}_2}b_{\mathbf{P}_1}$ is just the scattering between two excitons with the opposite orientation of dipole (the momenta change). After scattering both magnetoexcitons stay in their initial layers.

Let us consider the temperature $T=0$. Assuming the majority of particles are in the condensate $[(N-N_0)/N_0\ll 1$, where N and N_0 are the total number of particles and the number of the particles in condensate, respectively], we account as in Bogoliubov approximation only the interaction between condensate particles and excited particles with condensate particles, and neglect the interaction between noncondensate particles. Then the total Hamiltonian $\hat{H}_{\rm tot}$ transforms to

$$\hat{H}_{\rm tot}=\frac{1}{2}\sum_{\mathbf{P}\neq 0}\{[\varepsilon_0(p)+(U_++U_-)n](a^+_{\mathbf{P}}a_{\mathbf{P}}+b^+_{\mathbf{P}}b_{\mathbf{P}}+a^+_{-\mathbf{P}}a_{-\mathbf{P}} + b^+_{-\mathbf{P}}b_{-\mathbf{P}})+2U_+n(a^+_{\mathbf{P}}a^+_{-\mathbf{P}}+a_{\mathbf{P}}a_{-\mathbf{P}}+b^+_{-\mathbf{P}}b^+_{\mathbf{P}}+b_{-\mathbf{P}}b_{\mathbf{P}}) + U_-n(a^+_{\mathbf{P}}b^+_{-\mathbf{P}}+a_{\mathbf{P}}b_{-\mathbf{P}}+a^+_{-\mathbf{P}}b^+_{\mathbf{P}}+a_{-\mathbf{P}}b_{\mathbf{P}}+a^+_{\mathbf{P}}b_{\mathbf{P}}+a^+_{-\mathbf{P}}b_{-\mathbf{P}} + a_{\mathbf{P}}b^+_{\mathbf{P}}+a_{-\mathbf{P}}b^+_{-\mathbf{P}})\}. \tag{19}$$

Let us diagonalize Hamiltonian $\hat{H}_{\rm tot}$ by using the Bogoliubov-type unitary transformation[35]

$$a_{\mathbf{P}}=\frac{1}{\sqrt{1-A^2_{\mathbf{P}}-B^2_{\mathbf{P}}-C^2_{\mathbf{P}}}}(\alpha_{\mathbf{P}}+A_{\mathbf{P}}\alpha^+_{-\mathbf{P}}+B_{\mathbf{P}}\beta^+_{-\mathbf{P}}+C_{\mathbf{P}}\beta_{\mathbf{P}}),$$

$$b_{\mathbf{P}}=\frac{1}{\sqrt{1-A^2_{\mathbf{P}}-B^2_{\mathbf{P}}-C^2_{\mathbf{P}}}}(\beta_{\mathbf{P}}+A_{\mathbf{P}}\beta^+_{-\mathbf{P}}+B_{\mathbf{P}}\alpha^+_{-\mathbf{P}}+C_{\mathbf{P}}\alpha_{\mathbf{P}}), \tag{20}$$

where the coefficients $A_{\mathbf{P}}$, $B_{\mathbf{P}}$, and $C_{\mathbf{P}}$ are found from the condition of vanishing of the coefficients at nondiagonal terms in the Hamiltonian. As a result we obtain

$$\hat{H}_{\rm tot}=\sum_{\mathbf{P}\neq 0}\varepsilon(p)(\alpha^+_{\mathbf{P}}\alpha_{\mathbf{P}}+\beta^+_{\mathbf{P}}\beta_{\mathbf{P}}) \tag{21}$$

with the spectrum of quasiparticles $\varepsilon(P)$:

$$\varepsilon_1^2(P)=\varepsilon_0^2(P)+2nU_+\varepsilon_0(P),$$

$$\varepsilon_2^2(P)=\varepsilon_0^2(P)+2n(U_++U_-)\varepsilon_0(P). \tag{22}$$

Since $U_+>0$ and $U_-<0$, we have $\varepsilon_1^2(P)>\varepsilon_2^2(P)$ at $P>0$. Therefore, at low temperatures the quasiparticles only with the spectrum $\varepsilon_2^2(P)$ will be excited, since the excitation of these quasiparticles requires less energy than for the quasiparticles with the spectrum $\varepsilon_1^2(P)$. Since $U_++U_-<0$, it is easy to see from Eq. (22) that for the small momenta $P<\sqrt{4m_Bn|U_++U_-|}$ the spectrum of excitations becomes imaginary. Hence, the system of weakly interacting indirect magnetoexcitons in the slab of the superlattice is unstable. It can be seen that the condition of the instability of magnetoexcitons becomes stronger as magnetic field becomes higher, because m_B increases with the increment of the magnetic field, and, therefore, the region of P resulting in the imaginary collective spectrum increases as B increases.

The mathematically similar system was considered in Ref. 36, where 2D spinor polaritons were studied, with attractive interactions between the polaritons of the opposite circular polarization and repulsive interaction for polaritons with the same circular polarization. Note that our results for magnetoexcitonic spectrum given by Eq. (22) are analogous to that in Ref. 36. While in Ref. 36 the repulsion exceeds the attraction between two different polaritons ($|U_+|>|U_-|$), in multilayer superlattice with parallel and opposite dipoles there is an opposite case ($|U_+|<|U_-|$). Therefore, there is an instability in multilayer magnetoexciton system while there is no instability in polariton system considered in Ref. 36.

The analogous instability was studied theoretically in 2D atomic gas with the attractive and repulsive interaction between bosons.[38]

IV. SUPERFLUIDITY OF QUADRUPOLE MAGNETOBIEXCITONS IN QW AND GRAPHENE SUPERLATTICES

Let us consider the low-density weakly nonideal gas of two-dimensional indirect magnetobiexcitons, created by the indirect magnetoexcitons with opposite dipoles in the neighboring pairs of wells as the ground state of the system (Fig. 1). The ratio of the magnetobiexciton and magnetoexciton energies, as well as the ratio of the magnetoexciton and magnetobiexciton radii along the QWs or GLs, is the parameter for the adiabatic approximation used in Ref. 39. These parameters are small and they are even smaller than analogous parameters for atoms and molecules. The smallness of these parameters will be verified below by the calculations of the corresponding energy and radius of the indirect magnetobiexciton. Here it was assumed that the distance D between the QWs or GLs is greater than the radius of indirect magnetobiexciton r_0, $D>r_0$. At $r>1.11D$ indirect magnetoexcitons attract and at $r<1.11D$ they repel. The minimum of the potential energy $U(r)$ locates at $r=r_0\approx 1.67D$ between the indirect excitons and this r_0 is the mean radius of the magnetobiexciton along wells or graphene layers. At large D one can expand the potential energy $U(r)$ in terms of the parameter $(r-r_0)/D\ll 1$:

$$U(r) = -0.04\frac{e^2}{\epsilon D} + 0.44\frac{e^2}{\epsilon D^3}(r-r_0)^2 + \dots . \tag{23}$$

If we restrict ourselves by the first two terms of this expansion it easy to see that at large D magnetobiexciton levels correspond to the two-dimensional harmonic oscillator with the frequency $\omega=0.88e^2/(m_B\epsilon D^3)$:

$$E_n = -0.04\frac{e^2}{\epsilon D} + 2\sqrt{2}E_0\left(\frac{r^*}{D}\right)^{3/2}(n+1), \tag{24}$$

where $E_0=m_B\tilde{e}^4/(\hbar^2\epsilon)$, $r^*=\hbar^2\epsilon/(2m_B\tilde{e}^2)$, and $\tilde{e}^2=0.88e^2$. In the ground state the characteristic spread of the magnetobiexciton a_b along the QWs or GLs near the mean radius r_0 is

$$a_b = \sqrt{\frac{2\hbar}{m_B\omega}} = (8r^*)^{1/4}D^{3/4} = 1.03a_{ex}, \tag{25}$$

where $r_{ex}=(8r_{ex})^{1/4}D^{3/4}$ and $a_{ex}=\hbar^2\epsilon/(2m_Be^2)$ are the radius magnetoexciton and the two-dimensional effective Bohr radius with the effective magnetic mass m_B, respectively. Hence, the ratio of the binding energies of the magnetobiexciton $E_{\rm bex}$ and magnetoexciton E_{ex} is $E_{\rm bex}/E_{ex}=0.04\ll 1$ at $D\gg r_{ex}$, and the ratio of radii of the magnetoexciton and magnetobiexciton is $r_0/r_{ex}=0.67(8a_{ex})^{1/4}D^{-1/4}\ll 1$. Thus, the adiabatic condition is valid.

The mean dipole moment of the indirect magnetobiexciton equals to zero. However, the quadrupole moment is nonzero and equal to $Q=3eD^2$, and the large axis of the quadrupole is normal to the quantum wells or graphene layers. Hence, the indirect magnetobiexcitons interact at long distances $R\gg D$ as parallel quadrupoles: $U(R)=9e^2D^4/(\epsilon R^5)$.

We account the scattering of the magnetobiexciton on magnetobiexciton by using the results of the theory of two-dimensional Bose gas.[5] The chemical potential μ for two-dimensional biexcitons, repelled by the quadrupole law, in the ladder approximation, has the form (compare to Refs. 5 and 9)

$$\mu = \frac{2\pi\hbar^2 n_b}{m_B \log\{\hbar^{4/3}\epsilon^{2/3}/[8\pi(18m_Be^2D^4)^{2/3}n_b]\}}. \tag{26}$$

where $n_b=n/(2s)$ is the density of magnetobiexcitons in the QWs and $n_b=n/(8s)$ in the GLs. The chemical potential [Eq. (26)] is different than the chemical potential for two-dimensional excitons with the dipole-dipole repulsion given in Refs. 5 and 9).

At small momenta the collective spectrum of the magnetobiexciton system is the soundlike $\varepsilon(p)=c_sp$, where $c_s=\sqrt{\mu/(2m_B)}$ is the sound velocity, and satisfied by the Landau criterion for superfluidity. The density of the superfluid component $n_S(T)$ for the two-dimensional system with this sound spectrum can be estimated as[37]

$$n_S(T) = n_b - \frac{3\zeta(3)}{4\pi\hbar^2}\frac{k_B^3T^3}{m_Bc_s^4}, \tag{27}$$

where $\zeta(z)$ is the Riemann zeta function, and $\zeta(3)\simeq 1.202$. The second term in Eq. (27) is the temperature-dependent normal density taking into account the gas of phonons ("bogolons") with the dispersion law $\varepsilon(p)=\sqrt{\mu/(2m_B)}p$, where μ is given by Eq. (26).

In a 2D system, superfluidity of magnetobiexcitons appears below the Kosterlitz-Thouless transition temperature $T_c=\pi n_S(T)/(4m_B)$, where only coupled vortices are presented.[26] Employing $n_S(T)$ for the superfluid component, we obtain the cubic equation for the Kosterlitz-Thouless transition temperature T_c, which has the following solution:

$$T_c = \left[\left(1+\sqrt{\frac{32}{27}\left(\frac{8m_Bk_BT_c^0}{\pi\hbar^2n_b}\right)^3+1}\right)^{1/3} - \left(\sqrt{\frac{32}{27}\left(\frac{8m_Bk_BT_c^0}{\pi\hbar^2n_b}\right)^3+1}-1\right)^{1/3}\right]\frac{T_c^0}{2^{1/3}}. \tag{28}$$

Here, T_c^0 is an auxiliary quantity, equal to the temperature at which the superfluid density vanishes in the mean-field approximation, i.e., $n_S(T_c^0)=0$, $T_c^0=k_B^{-1}\{4\pi\hbar^2nc_s^4m_B/[3\zeta(3)]\}^{1/3}$. The temperature $T_c^0=T_c^0(B,D)$ may be used to estimate the crossover region where the local superfluid density appears

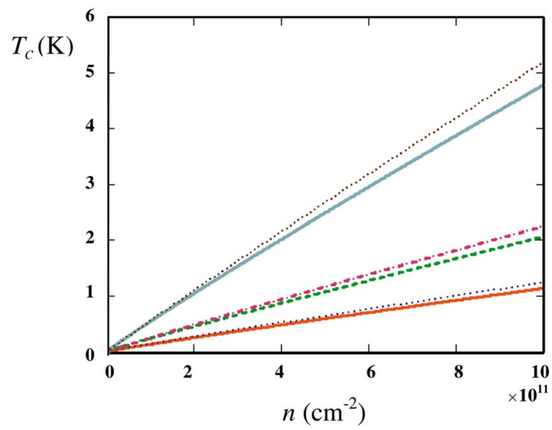

FIG. 3. (Color online) Dependence of the Kosterlitz-Thouless transition temperature $T_c=T_c(B)$ for the superlattice consisting of the QWs for GaAs/AlGaAs, $\epsilon=13$; and for GLs separated by the layer of SiO_2 with $\epsilon=4.5$ on the magnetoexciton density n at $D=10$ nm at different magnetic fields. The solid, dashed, and thin solid curves for the QWs, dotted, dashed-dotted, and thin dotted curves for the GL at B: $B=20$ T, $B=15$ T, and $B=10$ T, respectively.

for magnetoexcitons on a scale smaller or of the order of the mean intervortex separation in the system. The local superfluid density can manifest itself in the local optical or transport properties.

V. RESULTS AND DISCUSSION

We calculated the dependence of the Kosterlitz-Thouless transition temperature T_c on the density of the magnetoexcitons for the superlattice consisting of the quantum wells and graphene layers. The results of the calculations for the transition temperature T_c as a function of the density of the magnetoexcitons for the different values of the magnetic field B for the QWs and GLs are presented in Fig. 3. Analysis of these results shows that, firstly, for the given thickness of the QWs or GLs and for the fixed magnetic field the Kosterlitz-Thouless transition temperature T_c in good approximation linearly increases with the increase in the magnetoexcitons density for the QWs, as well as for GLs. This is due to the fact that the denominator in Eq. (26) for the chemical potential and, therefore, the sound velocity, weakly depends on n. Our calculations show that the slope of the function $T_c(n)$ decreases with n very weakly and it is almost constant. Secondly, for the same magnetoexcitons density the transition temperature T_c for the GLs is always higher than for the QWs, because the magnetic mass of magnetoexciton in the GLs is four times smaller than in the CQWs due to the four-component spinor structure of the magnetoexciton wave function in the GLs. Moreover, the transition temperature strongly depends on the magnetic field: with the increase in the magnetic field the transition temperature T_c dramatically increases for the GLs, as well as for the QWs. This is due to

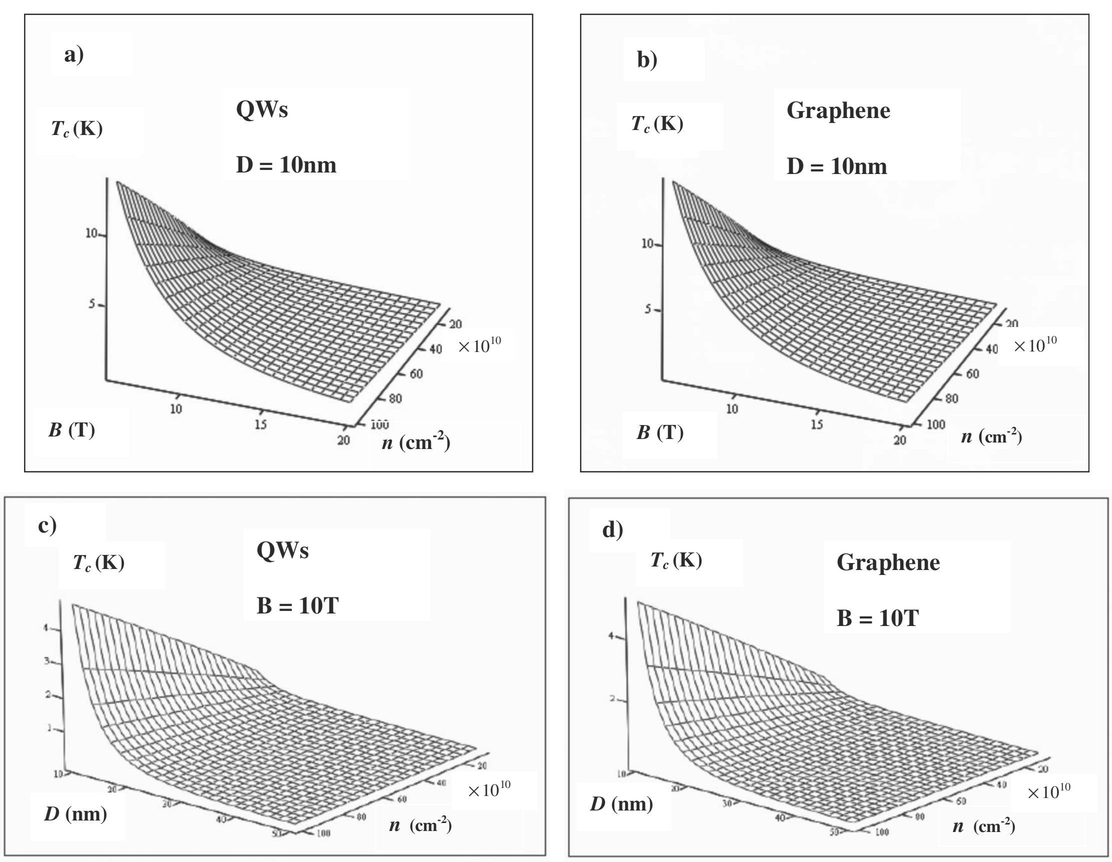

FIG. 4. Dependence of the Kosterlitz-Thouless transition temperature $T_c=T_c(B)$ for the superlattice consisting of the QWs (for GaAs/AlGaAs $\epsilon=13$) and of the GLs (for GLs separated by the layer of SiO_2 $\epsilon=4.5$) on the magnetoexciton density n and magnetic fields B presented by (a) and (b) and on the magnetoexciton density n and the interlayer separation D presented by (c) and (d), respectively.

the increment of m_B as a function of B and D. The T_c decreases as $B^{-1/2}$ at $D \ll r_B$ and as B^{-2} when $D \gg r_B$. The three-dimensional plots of the dependence of the Kosterlitz-Thouless transition temperature T_c on the magnetic field and magnetoexcitons density for the superlattice consisting from the QWs and GLs are shown in Fig. 4.

The advantage of the observation of the magnetoexciton superfluidity and the BEC in graphene in comparison with this in the CQWs consists of essentially weak influence of the random field on T_c due to the fact that the density of defects in graphene is sufficiently lower than in the CQWs (due to the absence of the roughness of the QWs boundaries). As known, the influence of impurities is suppressed due to the existence of the Berry phase.[40] Particularly, Berry phase makes impossible backscattering of the excitons on the impurities. Due to this fact, the influence of impurities on the BEC occurs to be smaller than for the semiconductor QWs. The disorder more likely suppresses the superfluidity and the Kosterlitz-Thouless temperature for magnetobiexcitons in the QW superlattices analogously to the case of magnetoexcitons in the CQWs.[41]

VI. CONCLUSIONS

It is shown that the low-density system of the indirect magnetoexcitons in a slab of the superlattice of the second type or consisting of the QWs or GLs in high magnetic field occur to be *instable* due to the attraction of magnetoexcitons with opposite dipoles at large distances. The instability of the ground state of the system of interacting two-dimensional indirect magnetoexcitons in a slab of superlattice with the alternating electron and hole layers for both QWs and GLs in high magnetic field is claimed. This instability is due to the attraction between the indirect excitons with the opposite directed dipole moments. The stable system of indirect quasi-two-dimensional magnetobiexcitons, consisting from the indirect excitons with opposite directed dipole moments in the superlattices, is *stable* due to the quadrupole-quadrupole repulsion. Therefore, at the pumping increase at low temperatures the excitonic line must vanish and only the magnetobiexcitonic line survives. Note that in spite of both the QW and graphene realizations represented by completely different Hamiltonians, the effective Hamiltonian in a strong magnetic field was obtained to be the same. The projection of the electron-hole Hamiltonian for the CQWs [Eq. (2)] and GLs [Eq. (8)] in magnetic field onto the lowest Landau level leads to the effective Hamiltonian [Eq. (14)] without the vector potential and renormalized effective mass of the magnetoexciton, which depends on the magnetic field. The magnetic field in the effective Hamiltonian [Eq. (14)] is presented into the renormalized mass m_B of the magnetoexciton. Moreover, for N excitons we have reduced the number of the degrees of freedom from $2N \times 2$ to $N \times 2$ by integrating over the coordinates of the relative motion of the electrons and holes. The Kosterlitz-Thouless transition to the superfluid state is calculated for the system of indirect magnetobiexcitons. The temperature T_c for the onset of superfluidity due to the Kosterlitz-Thouless transition at *a fixed magnetobiexciton density* decreases as a function of magnetic field B and interlayer separation D, and almost linearly increases when the magnetoexcitons density of the QWs or GLs increases. At the same exciton density n the Kosterlitz-Thouless temperature T_c is higher for the superlattice consisting of the GLs than for the superlattice consisting of the quantum wells.

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